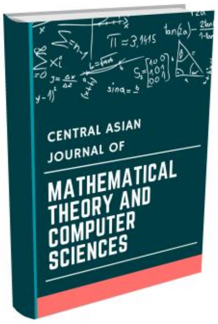

CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES

https://cajmtcs.centralasianstudies.org

Volume: 04 Issue: 03 | Mar 2023 **ISSN: 2660-5309**

APPLICATION FOR LINEAR PROGRAMMING TO SOLVE OPTIMIZATION PROBLEMS

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Annotation

To assess the degree of practical implementation of the optimization principle when comparing options for plans compiled in different ways, this paper proposes a comprehensive indicator of the effectiveness of planned calculations. The advantage of the indicator is that its value is proportional to the magnitude of potential losses from incomplete and incomplete use of available resources, that is, those factors that symbolize the loss of resources in the economic planning process, but have not yet served as criteria for the quality of planning decisions. Therefore, the fact that the optimization method makes it possible to improve (reduce) the value of these indicators with the same volumes of available production resources allows us to conclude that the structural optimization method is very effective and promising in solving production problems of linear programming and in the process of economic planning.

ARTICLEINFO

Article history:
Received 28 Jan 2023
Revised form 25 Feb 2023
Accepted 29 Mar 2023

Key words: *optimum choice, optimization models, perfect solution, Mathematical optimizer model.*

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Introduction:

In the 18th century, the mathematical foundations of optimization were already laid (calculus, numerical methods, etc.). However, until the second half of the 20th century, optimization methods were rarely used in many areas of science and technology, since the practical use of mathematical optimization methods required a large amount of computational work, which was very difficult to do without a computer, and in some cases not at all. The statement of the optimization problem assumes the presence of competing characteristics of the process, for example:

- Production volume Raw material consumption
- quantity of goods
- quality of goods
- Choosing a normalization option for given properties is an action to solve an optimization problem.

Linear programming is one of the first and most complete branches of mathematical programming. Linear programming was the branch from which the discipline "mathematical programming" began to develop. The term "programming" in the name of the discipline has nothing to do with the term "programming (i.e. writing programs) for computers", since the discipline "linear programming" arose even before the time when computers were widely used to solve mathematical, engineering and economic tasks. problems and other issues[1]. The term "linear programming" arose as a result of an inaccurate translation from the English language "linear programming". One of the meanings of the word “programming” is planning and planning. Therefore, the correct translation of "linear programming" will not be "linear programming", but "linear planning", which more accurately reflects the content of the discipline. However, the term linear programming, non-linear programming, etc. In our literature, this has become commonplace. Therefore, linear programming arose after the Second World War and began to develop rapidly, attracting the attention of mathematicians, economists and engineers with the possibility of wide practicality, as well as mathematical "harmony". We can say that linear programming is applicable to building mathematical models of those processes that can be based on the premise of a linear representation of the real world: economic problems, management and planning problems, optimal equipment placement[2].

Linear programming problems are called problems in which both the objective function and the constraints in the form of equality and inequality are linear. Briefly, the linear programming problem can be formulated as follows: find a vector of variable values that represents the maximum linear objective function under the constraints m in the form of a linear equality or inequality. Linear programming is the most widely used optimization technique. Linear programming tasks include:

- rational use of raw materials and materials.
- reduction of improvement tasks.
- optimization of the production program of enterprises.
- optimal location and concentration of production.
- Develop an ideal transportation plan and the transportation process itself.
- Inventory Management.

So, according to American experts, about 75% of the total number of optimization methods used is linear programming. About a quarter of the time spent by a computer in recent years has been devoted to scientific research on solving linear programming problems and their many modifications.

The first statements of linear programming problems were formulated by the famous Soviet mathematician L.V. Kantorovich, who received the Nobel Prize in Economics for this work.

The theory and computational apparatus of linear programming received significant development with the invention and spread of computers and the formulation of the American mathematician J. Danzig for a simple method.

Currently, linear programming is one of the most widely used mathematical theory tools for optimal decision making. To solve linear programming problems, complex programs have been developed that allow solving large-scale practical problems efficiently and reliably. These programs and systems are equipped with advanced systems for preparing initial data and tools for their analysis and display of results.

The work and talents of many mathematicians have been invested in the development and improvement of these systems, and experience has been gained in solving thousands of problems. Proficiency with a linear programming device is necessary for every specialist in mathematical programming. Linear programming is closely related to other methods of mathematical programming. The division of optimization problems into these categories is of great importance, since the features of some problems play an important role in the development of methods for solving them[3].

1. USE OF LINEAR PROGRAMMING FOR SOLVING OPTIMIZATION PROBLEMS

1.1 General optimization problem

Optimization as a branch of mathematics has been around for a long time and refers to choice, that is, to what you have to do constantly in everyday life. The term "optimization" in the literature refers to a process or series of operations that produces an accurate solution. Although the ultimate goal of optimization is to find the best or "optimal" solution, one should be content with improving rather than perfecting known solutions. Therefore, improvement should rather be understood as striving for perfection, which will probably not be achieved.

Practice generates more and more optimization problems, and its complexity increases. New mathematical models and methods are required that take into account the presence of many criteria and conduct a global search for optimal ones. In other words, life forces us to develop a sports apparatus for improvement.

Real applied optimization problems are very complex. Modern optimization methods do not always deal with solving real problems without human help. No, there is no theory yet that would take into account any features of the functions that describe the problem statement. Preference should be given to methods that are easy to manage in the process of solving the problem.

Imagine that all managerial decisions are made in the best possible way. That is, all the parameters that the firm can influence are optimal. Then the company will receive the maximum profit (it is impossible to get more under these conditions)[4]. In order to determine how the company's employees make optimal management decisions, mathematical programming methods can be used.

In economics, optimization problems arise in connection with several possible options for the functioning of a particular economic object, when there is a situation of choosing the best option according to a certain rule, a criterion characterized by an appropriate objective function (for example, to obtain the minimum cost, the maximum volume of production).

In general, the mathematical formulation of the problem of mathematical programming consists in determining the largest or smallest value of the objective function $f(x_1, x_2, ..., x_n)$ under the conditions $g_i(x_1, x_2, ..., x_n)$ b_i; (i =1,2,…m), where f and g_i; are given functions and bi are some real numbers.

Mathematical programming problems are divided into linear and non-linear programming problems. if all functions f and g_i are linear, then the corresponding problem is a linear programming problem. If at least one of these functions is non-linear, then the corresponding problem is a non-linear programming problem.

1.2 Statement of optimization problems

When setting an optimization problem, it is necessary to:

1. The presence of an optimization object and optimization goals. At the same time, the formulation of each optimization problem should require an extremal value of only one quantity, i.e. At the same time, two or more optimization criteria should not be assigned to the system.

A typical example of an incorrect formulation of an optimization problem:

"Get the highest performance at the lowest cost."

The error lies in the fact that the task is to find the optimality of 2 quantities that contradict each other in their essence.

The correct statement of the problem could be as follows:

a) get the maximum performance at a given cost;

b) get the minimum cost at a given performance;

In the first case, the optimization criterion is productivity, and in the second, the cost.

2. The presence of optimization resources, which is understood as the possibility of choosing the values of some parameters of the object being optimized.

3. The possibility of quantifying the value being optimized, since only in this case it is possible to compare the effects of the choice of certain control actions[5].

4. Consideration of restrictions.

Usually, the value to be optimized is related to the efficiency of the object under consideration (apparatus, workshop, plant). The optimized version of the operation of the object must be evaluated by some quantitative measure - the criterion of optimality.

The optimality criterion is a quantitative assessment of the quality of the object being optimized.

Depending on its formulation, any of the optimization problems can be solved by various methods, and vice versa - any method can be used to solve many problems. Optimization methods can be scalar (optimization is carried out according to one criterion), vector (optimization is carried out according to many criteria), search methods (include methods of regular and random search methods), analytical (methods of differential calculus, methods of variational calculus, etc.), computational (based on on mathematical programming, which can be linear, non-linear, discrete, dynamic, stochastic, heuristic, etc.), probabilistic, game-theoretic, etc. Problems can be subjected to optimization both with and without restrictions.

2. MODELS OF LINEAR PROGRAMMING

2.1 Statement of the problem of linear programming

Linear optimization problems are a type of extremal problems formed by linear functions and linear relations. One way or another, the basic problem of this kind is a linear programming problem, i.e. the problem of finding an extremum (maximum or minimum) of a linear function under constraints in the form of linear inequalities. The enormous interest in such problems was determined by their economic content in time, this is the beginning and especially the end of the 30s, and then until the 50s of the XX century. and continues to the present[6].

Linear programming is an integral part of the branch of mathematics that studies methods for finding the conditional extremum of a function of many variables and is called mathematical programming. In classical mathematical analysis, the problem of finding the conditional extremum of a function is considered. However, time has shown that for many tasks that arise under the influence of practical demands, classical methods are insufficient. In connection with the development of technology, the growth of industrial production and with the advent of computers, the tasks of finding optimal solutions in various spheres of human activity began to play an increasingly important role. The main tool in solving these problems was mathematical modeling - a formal description of the phenomenon under study and research using mathematical apparatus.

Linear programming models are used to find the optimal solution in a situation of allocation of scarce resources in the presence of competing needs. For example, using a linear programming model, a production manager can determine the optimal production program, i.e. calculate how many products of each item should be produced to obtain the greatest profit with known volumes of materials and parts, the fund of equipment operation time and the profitability of each type of product. Most of the optimization models developed for practical application are reduced to linear programming problems. However, taking into account the nature of the analyzed operations and the established forms of the dependence of factors, models of other types can also be used. With non-linear forms of the dependence of the result of the operation on new factors - non-linear programming models; if it is necessary to include the time factor in the analysis - dynamic programming models; with the probabilistic influence of factors on the result of the operation - models of mathematical statistics.

Linear programming deals with the optimization of models in which the objective function depends linearly on the decision variables. Constraints are also linear inequalities or equations in decision variables. The linearity requirement means that both the objective function and the constraints can only be sums of products of constant coefficients and solution variables. The art of mathematical modeling is to take into account as many factors as possible by simple means. This is why the modeling process is often iterative. At the first stage, a relatively simple model is built and its study is carried out, which makes it possible to understand which of the essential properties of the object under study are not captured by this formal scheme. Then there is a refinement, complication of the model.

In most cases, the first degree of approximation to reality is a model in which all dependencies between variables characterizing the state of an object are assumed to be linear. Here there is a complete analogy with how very important and often exhaustive information about the behavior of an arbitrary function is obtained on the basis of studying its derivative - this function is replaced in the neighborhood of each point with a linear dependence. A significant number of economic, technical and other processes are quite well and fully described by linear models[7].

3. Reducing the problem to a standard form

To reduce this problem to a standard form, it is only necessary to move from restrictions - inequalities to equalities. To do this, we introduce additional balance sheet non-negative variables. Also, to simplify further calculations, we divide both parts of the restrictions on the assembly of parts by 5:

X1 + X2 + X3 + X7 = 8;

X4 + X5 + X6 + X8 = 8;

2X1 - X2 + 6X4 - 3X5 = 0;

2X1 - 2X3 + 6X4 - 2X6 =0;

X1 , X2 , X3 , X4 , X5 , X6 , X7 , X8 ? 0.

E= X1 + X2 + 2X3 + 3X4 + 3X5 + 2X6 ?max

where X7, X8 are residual variables.

So, we have reduced our original problem to the standard form of the main problem of linear programming.

For a problem presented in standard form, the number of variables is usually greater than the number of constraints. Therefore, to find the initial solution of the problem, it is required to express m variables (i.e., the number of variables equal to the number of equations) in terms of the remaining n-m variables, take these n-m variables equal to zero and, thus, find the values of m variables (in the given problem, m = 4 and n=8). Variables whose values are taken equal to zero are called non-basic, and the remaining m variables are called basic. The values of the basic variables are non-negative (some of them may be equal to zero). The number of basic variables is always equal to the number of constraints. The solution found in this way is called the initial admissible basic solution. It complies with all restrictions.

The initial solution is easiest to find in the case when each constraint has a variable that enters it with a coefficient of 1 and is absent from other constraints. Such variables are taken as basic ones (they form the initial basis of the problem). The remaining (non-basic) variables are taken equal to zero. Thus, the basic variables take values equal to the right parts of the restrictions[8].

So, to find the initial feasible solution, it is necessary that each of the equations includes a variable with a coefficient of 1 and does not enter into other equations (basic variable). In our case, we have only 2 basic variables (X7 and X8), two more basic variables are missing. They can be created using a special method called artificial basis building.

4. Construction of an artificial basis

Artificial basis methods are designed to construct an initial basis (i.e., to obtain an initial solution) in cases where its construction directly on the basis of the standard form is impossible. When using an artificial basis, the initial solution turns out to be invalid; from it, according to certain algorithms, a transition is made to the initial feasible solution.

In order to build an artificial basis, it is necessary to add one artificial variable to each equation of the standard form that does not contain basic variables (that is, obtained from an equality constraint or "not less"). In our case, this is:

2X1 - X2 + 6X4 - 3X5 + X9 = 0;

2X1 - 2X3 + 6X4 - 2X6 + X10 = 0.

where X9 and X10 are artificial variables that have no physical meaning, and X9, X10 ≈ 0.

After constructing an artificial basis, giving zero values to all variables, except for the basic ones, we get the initial basis: X7, X8, X9, X10. In total, there are four variables in the basis and their values are equal to the right parts of the restrictions, i.e.:

X7 = 8;

X8 = 8;

X9 = 0;

X10 = 0.

Now it is necessary to solve this problem, i.e. find the optimal feasible solution. To do this, we use the two-stage simplex method.

3.3 First step of the two-step simplex method

linear programming problem optimization

So, at the first stage of the two-stage method, an initial admissible solution is found. To do this, perform the following steps:

1. We build an artificial objective function - the sum of all artificial variables:

W = X9 + X10 ? min

2. Since the objective function must be expressed only in terms of non-basic variables, we express the artificial variables X9 and X10 in terms of non-basic variables, and then, having simplified the resulting expression, we rewrite the artificial objective function:

X9 = - 2X1 + X2 - 6X4 + 3X5;

X10 = - 2X1 + 2X3 - 6X4 + 2X6.

W = - 4X1 + X2 + 2X3 - 12X4 + 3X5 + 2X6 ? min

3. To bring it to the standard form, let's direct the artificial objective function to the maximum, for this we multiply both parts of it by -1:

-W = 4X1 - X2 - 2X3 + 12X4 - 3X5 - 2X6 ? max

4. We determine the initial, unacceptable solution. The basis consists of four variables, two of them are artificial, the other two are residual. Basic variables take values equal to the constraints of the problem. The rest of the variables are assumed to be zero. In this case, the objective function E takes the value 0, the artificial objective function -W also takes the value 0.

5. Compile the original simplex table:

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	
E	-1	-1	-2	-3	-3	-2	0	0	0	0	0

-W	-4	1	2	-12	3	2	0	0	0	0	0
X7	1	1	1	0	0	0	1	0	0	0	8
X8	0	0	0	1	1	1	0	1	0	0	8
X9	2	-1	0	6	-3	0	0	0	1	0	0
X10	2	0	-2	6	0	-2	0	0	0	1	0

Table 1. Simplex table

So, in the first column of the table, the basic variables are indicated, in the last column - their values, as well as the values of the objective and artificial objective functions. The table header lists all variables used. The rows of the table indicate the coefficients of the problem constraints.

6. We implement the first stage of the two-stage method: using the procedures of the simplex method, we perform the maximization of the -W function. In this case, the variables included in the basis are selected according to the W-row (i.e., on each cycle, the basis includes the variable that corresponds to the negative element in the W-row with the maximum absolute value; the column corresponding to this variable becomes the leading one). In our case, this is the X4 column, since the coefficient for this variable in the W-row is -12. We define the leading row as follows: we calculate the so-called simplex ratios, i.e., the ratios of the current values of the basic variables to the positive coefficients of the leading column corresponding to these basic variables. Then we take the minimum of these ratios and, by which line it corresponds, we determine the leading line. We have three such relationships: x8 (8/1=8), x9(0/6=0) and x10 (0/6=0). It turned out two minimum values, so we take any of them, for example, according to the variable X9. After we find the leading element, it is located at the intersection of the leading row and the leading column (in our case, it is 6). Then we define the variables that we will exclude from the basis and include in it. The variable to which the leading column corresponds will be included in the basis instead of the variable to which the leading row corresponds. Further, all transformations are performed according to the usual formulas of the simplex method or according to the "rectangle rule". The entire simplex table is subjected to transformations, including the E-row, W-row, and decision column. We get a new simplex table:

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	
E	0	-1,5	-2	0	-4,5	-2	0	0	0,5	0	0
-W	0	-1	2	0	-3	2	0	0	2	0	0
X7	1	1	1	0	0	0	1	0	0	0	8
X8	-0,33	0,17	0	0	1,5	1	0	1	-0,17	0	8
X4	0,33	-0,17	0	1	-0,5	0	0	0	0,17	0	0
X10	0	1	-2	0	3	-2	0	0	-1	1	0

Table 2. Simplex table No. 1.

We got a new solution (X7,X8,X4,X10)=(8,8,0,0). This solution is unacceptable, since the basis contains an artificial variable X10. Let's do another iteration. On the -W line, to include in the basis, select the variable X5 (because -3 is the maximum negative number in absolute value). Column X5 becomes leading. According to the minimum simplex ratio (8/1.5=5.33; 0/3=0), we select the variable X10 to exclude from the basis. The leading element is 3. After the recalculations, we get a new simplex table:

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	
E	0	0	-5	0	0	-5	0	0	-1	1,5	0
-W	0	0	0	0	0	0	0	0	1	1	0
X7	1	1	1	0	0	0	1	0	0	0	8
X8	-0,33	-0,33	1	0	0	2	0	1	0,33	-0,5	8
X4	0,33	0	-0,33	1	0	-0.33	0	0	0	0,17	0
X5	0	0,33	-0,67	0	1	-0,67	0	0	-0,33	0,3	

Table 3. Simplex table No. 2.

Conclusion

Summing up the results of the method of solving optimization problems, one should note the simplicity and versatility of its use when searching for optimal production plans or methods in conditions of limited production resources used. The paper also shows that any additional criteria for the effectiveness and importance of a product or technological methods for its production—objective coefficients in linear programming problems—are counterproductive to the criterion for effective use of given constraints. The objectivity of this hypothesis is confirmed not only by the examples given in this paper, but also by the fact of limited resources. Moreover, the growth of national wealth is provided only by the entrepreneurs who benefit from the increase in the scale of production at the basic costs of production, and not by their savings and liberation.

The given examples show that the simplified algorithm of the optimization method makes it possible to obtain sufficiently accurate solutions for any systems of linear inequalities and linear programming problems involving optimization using certain constraints. The basis for building mathematical models is, first of all, the correct choice of criteria for an economic problem (or any other process), in which the desired objective is expressed as a linear objective function, and the limitations of the process will be written as a system of linear equations or inequalities.

To date, the optimization method is the only instrument capable of ensuring the implementation of the principle of democratic centralization in production planning, that is, the conditions under which the improvement of the plans of individual producers will ensure the optimal development of social production.

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